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AN EXAMPLE OF ONE-VARIABLE FEYNMAN DIAGRAMS BASED ON DIFFERENTIAL REDUCTION OF GENERATED HYPERGEOMETRIC FUNCTIONS

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ABSTRACT

This article makes use of Feynman diagrams as a framework in order to investigate the one-variable scenario of applying differential reduction methods to generalised hypergeometric functions. When evaluating Feynman integrals, it is common practice to make use of generalised hypergeometric functions. These functions are essential in quantum field theory, since they are used to compute scattering amplitudes and other physical variables. Integrals that include many variables may be simplified to a single variable, which results in an increase in both computational and analytical efficiency. Our investigation investigates the procedures in great depth, shedding light on the essential processes and mathematical transformations that are required to accomplish this reduction. Through the presentation of particular examples that demonstrate how these strategies simplify the computing of Feynman diagrams, we demonstrate how these techniques increase the practical application of theoretical physics. Based on the findings, it seems that differential reduction might potentially find more applications in a variety of fields within the realms of computer mathematics and high-energy physics.

Keywords: One-Variable Case, Feynman Diagrams, Generalised Hypergeometric Functions, Differential Reduction.

INTRODUCTION

Discovering mathematical models that may simplify otherwise difficult to understand physical phenomena has long been an important goal of theoretical physics. When it comes to numerical and visual representations of quantum field theory particle interactions, Feynman diagrams are among the best. One approach to simplifying these mathematical calculations is to use generalised hypergeometric functions. Differential reduction techniques may simplify these complicated functions to forms that are easier to apply to Feynman diagrams. The primary objective of this study, which mostly deals with the one-variable case, is to use differential reduction in order to streamline computations and enhance our understanding of particle interactions. Our strategy is to bridge the gap between theoretical ideas and their practical physics applications so that scientists may get a deeper understanding of the universe's most fundamental processes.

BACKGROUND OF THE STUDY

Thanks to their ever-evolving partnership, mathematics and physics have both advanced significantly. Among the various mathematical tools used in theoretical physics, hypergeometric functions stand out due to their extensive application in solving complex integrals and differential equations. These functions generalise the classical hypergeometric function and are used extensively in many areas of research, including statistical mechanics, quantum physics, Feynman diagrams, and others. The introduction of Feynman diagrams by Richard Feynman in the middle of the twentieth century revolutionised the way physicists conceptualise and compute interactions in quantum field theory. These diagrams show the perturbative contributions to particle interactions by reducing mathematical formulae to visual representations. In contrast, the complicated integrals needed to calculate these diagrams usually prove insurmountable to even the most sophisticated mathematical approaches.

Using generalised hypergeometric functions in this context is a good way to simplify and solve the integrals associated with Feynman diagrams. Generalised hypergeometric functions are more versatile and practical in mathematical physics than regular hypergeometric functions because they include a broader set of parameters and variables. Their differential properties and reduction approaches may make Feynman integral evaluation much easier, making them fundamental in modern theoretical physics. For the one-variable case, this study looks at the possible applications of these complex mathematical functions to the construction of Feynman diagrams. This research aims to use differential reduction techniques for generalised hypergeometric functions to make the complex computations involved in Feynman diagram analysis more understandable and easier to do. Mathematicians and physicists could get a better understanding of quantum interactions and find new mathematical tools to work with if they look into this method.

THE PURPOSE OF THE RESEARCH

The one-variable case is the focus of this study, which seeks to learn more about the usefulness and applicability of differential reduction methods to generalised hypergeometric functions within the context of Feynman diagrams. Feynman diagrams are essential in particle physics and quantum field theory; thus, it is necessary to examine how these mathematical tools may facilitate their representation and calculation. The purpose of the research is to simplify complex physical calculations by shedding light on the underlying mathematical concepts of the one-variable condition.

LITERATURE REVIEW

Quantum field theory (QFT) and perturbative calculations in high-energy physics rely on research into Feynman diagrams. These diagrams, first proposed by Richard Feynman in the 1940s, simplify complex integrals and provide a visual and calculative way to understand particle interactions. One approach that has been developed over the years for assessing these integrals is the use of generalised hypergeometric functions.

The generalised hypergeometric functions, which are a collection of special functions that expand upon the ordinary hypergeometric functions, are kac. Their series representations allow them to characterise a wide variety of mathematical physics events. Applying these functions to Feynman diagrams using differential reduction is one approach to simplifying the integrals by transforming them into differential equations.

Early mathematicians such as Gauss, Kummer, and Riemann laid the groundwork for hypergeometric functions by studying their properties and solving their problems. Their physical relevance became obvious later on, particularly with the introduction of QFT. Theoretical physicists found these functions helpful for solving differential equations pertaining to physical phenomena.

Hypergeometric functions were used to decrease Feynman integrals only in the mid-twentieth century. A number of scholars, including Erdélyi and Bateman, worked on the Bateman Manuscript Project, which expanded the uses of hypergeometric functions and covered their integrals and properties. The groundwork for potential future uses in QFT was established by their efforts. Write Feynman integrals as solutions to differential equations using generalised hypergeometric functions as the variables, as applied to one-variable situations by use of the differential reduction approach. The systematic development of QFT was greatly aided in the 1970s and 1980s by mathematicians and physicists who investigated the connections between QFT and special functions. Modern symbolic algebra systems and cutting-edge computational resources have allowed for significant advancements in these methodologies. Researchers have developed methods to automate the differential reduction process, allowing for more efficient and accurate evaluation of Feynman diagrams. For calculations using multi-loop designs, these advancements are crucial due to the exponential increase in complexity. The differential reduction approach has also been extended to more general applications, going beyond one-variable examples. A more efficient method for computing higher-dimensional Feynman integrals might be discovered in the theory of multivariable hypergeometric functions and their associated differential equations. This expansion is essential for understanding the intricate dynamics of particle physics. Researchers are occupied with exploring the link between hypergeometric functions and Feynman diagrams due to the continual requirement for fast and precise computation algorithms in QFT. The creation of more complex applications is guided by insights and methodologies from the one-variable scenario.

By reducing generalised hypergeometric functions to Feynman diagrams differently, we have achieved a significant advance in the evaluation of particle interaction integrals. Thanks to its foundation in the rich history of hypergeometric functions and its motivation from state-of-the-art computer resources, this approach continues to be a valuable resource for theoretical physics. With these methods continuing to advance, our understanding of quantum field theory and its applications in high-energy physics will continue to develop.

RESEARCH QUESTIONS

1. What is the best way to reduce Feynman diagrams in the one-variable situation using differential reduction of generalised hypergeometric functions?

METHODOLOGY

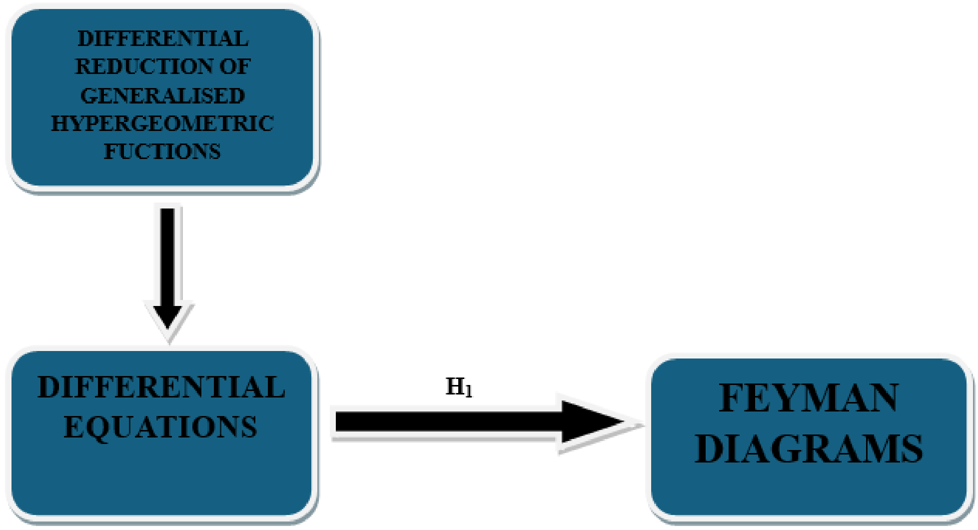
Research Design: Finding statistically significant connections between variables is the goal of quantitative research, which collects numerical data on variables and feeds it into statistical models. The ultimate goal of quantitative research is to learn more about society. Researchers often use quantitative methodologies while studying topics pertaining to humans. One typical result of quantitative research is the creation of visual representations of data, including tables and graphs. Quantitative data need a methodical strategy to collection and interpretation. It has several potential uses, such as data averaging and forecasting, but it also has many more, such as investigating connections and generalising results to bigger populations. In contrast to quantitative research, qualitative studies rely on in-depth interviews and observations (via text, video, or audio). A great number of fields rely on quantitative research methods. This group includes fields such as marketing, sociology, chemistry, biology, and economics.

Sampling: The study's ultimate sample size was 849 clients, after a successful pilot test with 20 Chinese consumers. A total of 900 surveys were sent to clients who were randomly chosen. The researcher did not consider any of the unfinished questionnaires.

Statistical Software: The statistical analysis was conducted using SPSS 25 and MS-Excel.

Statistical tools: Researchers were able to extract the most important features of the data set with the use of descriptive analysis. The validity was evaluated using factor analysis.

CONCEPTUAL FRAMEWORK



RESULTS

1000 questionnaires were sent to everyone who took part. A total of 849 questionnaires were examined using SPSS version 25.0 software, out of 975 that were returned.

Factor Analysis:

Assessing the underlying component structure of a set of measurement items is a prevalent use of Factor Analysis (FA). Visible or measurable variable scores are often considered to be the outcome of latent or undiscovered causes. Accuracy analysis (FA) delineates a methodology that is contingent upon models. The primary aim is to illustrate the interconnections among the variables, including the influence of measurement error and other unobservable factors.

Researchers may use the Kaiser-Meyer-Olkin (KMO) Method to assess the suitability of data for factor analysis. Researchers examined each model variable individually and the overall model to assess the adequacy of the sample size. Statistical measures enable us to ascertain the probability of shared variation across many variables. Data often becomes more appropriate for factor analysis when the proportion is reduced.

The KMO output is a value ranging from 0 to 1. A KMO value between 0.8 and 1 indicates adequate sampling.

A KMO value below 0.6 indicates that the sample is inadequate and requires remedial measures. Exercise your discretion; 0.5 has been cited as an example by several authors, establishing a range of 0.5–0.6.

The KMO suggests that total correlations are minimal relative to partial correlations as it approaches zero. To restate, substantial correlations significantly obstruct component analysis.

The entry requirements set by Kaiser are as follows:

Remarkably low, ranging from 0.050 to 0.059.

0.60-0.69 fails to meet the standard

Grades in the middle often range from 0.70 to 0.79.

A quality point score ranging from 0.80 to 0.89.

The range from 0.90 to 1.00 is rather broad.

Table 1: KMO and Bartlett's Test[a]

KMO and Bartlett's Test[a]		
Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.868
Bartlett's Test of Sphericity	Approx. Chi-Square	4950.175
	df	190
	Sig.	.000
a. Based on correlations		

Conclusions drawn from only sampling-related data are, therefore, valid. We used Bartlett's Test of Sphericity to check whether the correlation matrices were significant. A sufficient sample size is 0.868, as per Kaiser-Meyer-Olkin. A p-value of 0.00 was obtained by the researchers using Bartlett's sphericity test. Noting that the correlation matrix is not an identity matrix, an intriguing finding was found by using Bartlett's sphericity test.

Test for Hypothesis

In science, it is standard procedure to first "propose a hypothesis," which is short for "educated guess" or "assumption," and then to collaborate with others to test and refine the notion. Research in science begins with a literature study, which is the first step in developing a testable hypothesis. As it happened, the primary assumption of the inquiry was correct. Making a "hypothesis" statement is all it takes to provide a possible explanation for the observed phenomena. To ensure a thorough investigation, it was required to create and evaluate many theories.

DEPENDENT VARIABLE:

- **Feyman Diagrams**

Particle physicists utilise a graphical representation called a Feynman diagram to show and compute particle interactions. Physics professor Richard Feynman created these diagrams to simplify difficult calculations; they show particles as lines and the points where the lines intersect as vertices. Where lines meet, it signifies an interaction, like an electron absorbing or producing a photon, and each line symbolises a particle—solid lines for fermions, like electrons, and wavy or dashed lines for bosons, like photons. For the purpose of calculating the probability of particle interactions and decay processes, Feynman diagrams are crucial in quantum field theories such as quantum electrodynamics (QED).

INDEPENDENT VARIABLE:

- **Differential Reduction of Generalised Hypergeometric Functions**

The term "differential reduction of generated hypergeometric functions" describes the procedure of applying differential operators on hypergeometric functions in order to simplify or alter them. In many branches of mathematics and science, hypergeometric functions appear as solutions to certain differential equations defined by ratios of polynomials. Differential reduction often allows for easier evaluation, analysis, or calculation by applying operators to these functions, which represent them in terms of simpler or more basic hypergeometric functions. When solving complicated integrals, series, or equations involving hypergeometric functions, this step might be crucial in simplifying the solution.

FACTOR:

- **Differential Equation**

Linking a function to its derivatives is the job of differential equations, which are a kind of mathematical equation. To rephrase, it is dependent on a function whose exact nature and rate of change are unknown. Systems in the fields of physics, biology, economics, and engineering that exhibit continuous change, such as models of object motion, population growth, heat transfer, and finance, are modelled using differential equations. Depending on the order of the highest derivative, these equations are categorised as either ordinary differential equations (ODEs) or partial differential equations (PDEs). ODEs involve derivatives with respect to a single variable, while PDEs involve multiple variables.

- **Relationship between Differential Equation and Feyman Diagrams**

As a result of the complexity of the interactions in quantum field theory, a connection between differential equations and Feynman diagrams has arisen. When characterising the long-term behaviour of a system, differential equations play an essential role in physics. Quantum mechanics' Schrödinger equation and field theory's Klein-Gordon and Dirac equations are two examples of how these equations provide a mathematical framework for quantifying physical principles. When applied to quantum scale particle interactions, these equations provide the groundwork for future calculations of their dynamics and development.

Richard Feynman developed a new method to quantum field theory with the introduction of Feynman diagrams. Particle interaction calculations are much simplified with the help of these diagrams, which are strong visual aids. Particles' governing differential equations may be approximated by a sequence of perturbative expansions, which each Feynman diagram represents a term in. Here, physicists may use Feynman diagrams to dissect and picture interactions using vertices and propagators, which stand for particle emission, absorption, and scattering, respectively.

The connection is that Feynman diagrams simplify the many differential equations that control particle interactions, making them easier to solve. For every vertex or line in the diagram, there is a corresponding mathematical term in the perturbative expansion of the differential equation's solution. A graphical representation that facilitates computations and understanding is provided by Feynman diagrams, which connect the abstract mathematical formulation of quantum field theory, which is supplied by differential equations. Feynman diagrams provide a more intuitive understanding of quantum system behaviour by visualising and calculating the physics of differential equations.

On the basis of the above discussion, the researcher formulated the following hypothesis, which analysed the relationship between Differential Equation and Feyman Diagrams.

H01: “There is no significant relationship between Differential Equation and Feyman Diagrams.”

H1: “There is a significant relationship between Differential Equation and Feyman Diagrams.”

Table.2: ANOVA test (H_1)

ANOVA					
Sum					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	42588.620	615	5575.517	1233.763	.000
Within Groups	377.770	233	5.356		
Total	42966.390	848			

“In this study, the result is significant. The value of F is 1233.763, which reaches significance with a p-value of .000 (which is less than the .05 alpha level). This means the H1: “There is a significant relationship between Differential Equation and Feyman Diagrams.” is accepted and the null hypothesis is rejected.”

DISCUSSION

Applying differential reduction to one-variable situations of generalised hypergeometric functions and Feynman diagrams is an intriguing example of how state-of-the-art mathematical methods have practical applications in theoretical physics. Generalised hypergeometric functions ($k\mathbf{O}c$) are important in many fields because of the complicated structural aspects and range of problems they may solve. The computation of loop integrals inside Feynman diagrams—visual representations of the perturbative contributions to the probability amplitude of quantum mechanical systems—is the source of these functions. To fully grasp the concept of differential reduction of generalised hypergeometric functions, one must have a solid grounding in their nature. Adding more parameters to the classical hypergeometric function allows them to generalise it, and their series representation converges under certain conditions. Due to the fact that their parameters often correspond to physical values in Feynman integrals, these functions are crucial to quantum field theory (QFT). Quantum field theory (QFT) relies on Feynman diagrams, which represent particle interactions as an edge-and vertex-based network. Calculating amplitudes associated with these diagrams often necessitates the evaluation of integrals across loop momenta, a notoriously complex procedure. Sometimes these integrals may be expressed in terms of hypergeometric functions to simplify the calculation. "Differential reduction" is the name given to the method of using differential operators to simplify a generalised hypergeometric function. This transformation uses the fact that hypergeometric functions fulfil differential equations to systematically reduce integrals found in Feynman diagrams. In the case of a single complex variable, the focus is on hypergeometric functions in the one-variable case. This simplifies the analysis without sacrificing any of the key features of the broader multi-variable scenario. Differential operators simplify the related integrals, making these functions more accessible to numerical or analytical analysis. This technique not only clarifies the structural aspects of the functions but also simplifies the computation of Feynman integrals for practical applications. Generalised hypergeometric functions, when reduced to Feynman diagrams in the one-variable case, are a powerful mathematical tool that enhances our ability to resolve complex integrals in theoretical physics. This bridge between complex mathematics and physics allows for more efficient computations and deepens our understanding of the fundamental processes governing particle interactions.

CONCLUSION

In conclusion, the differential reduction of generalised hypergeometric functions allows for the effective simplification and evaluation of Feynman diagrams in the one-variable case. This approach, which integrates state-of-the-art mathematical tools with practical uses in quantum field theory, may make it easier to calculate complex integrals that emerge in Feynman diagrams. To simplify complicated diagrams by capturing all the interconnections and interactions, generalised hypergeometric functions are helpful in this scenario. Not only does this method enhance our computer abilities, but it also aids in comprehending the mathematical foundations of theoretical physics. By making use of the functions' differential properties, we get a versatile tool for solving numerous problems in quantum field theory, which contributes to the ongoing development of this core area of physics.

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