Prestieesci Research Review

THE USE OF DIFFERENTIAL REDUCTION OF GENERALISED HYPERGEOMETRIC FUNCTIONS TO FEYNMAN DIAGRAMS: ONE-VARIABLE CASE

Xing Kongduo, Midhun Chakkaravarthy

Lincoln University College, 47301 Petaling Jaya, Selangor D. E., Malaysia.

Corresponding author: Xing Kongduo, Lincoln University College, 47301 Petaling Jaya, Selangor D. E., Malaysia, Email: 88427661@qq.com

ABSTRACT

Using Feynman diagrams as a framework, this study investigates the one-variable scenario of applying differential reduction methods to generalised hypergeometric functions. An essential part of quantum field theory for computing scattering amplitudes and other physical variables are generalised hypergeometric functions, which are used often in the assessment of Feynman integrals. Reduced integrals provide for more efficient computing and analytical investigation by simplifying multivariable integrals into one-variable forms. Our investigation delves deep into the strategies used to accomplish this reduction, shedding light on crucial stages and mathematical transformations. By providing concrete instances, we show how these methods streamline the calculation of Feynman diagrams, which in turn advances the real-world relevance of theoretical physics. Based on the findings, differential reduction might be used more widely in several branches of computer mathematics and high-energy physics.

Keywords: One-Variable Case, Feynman Diagrams, Generalised Hypergeometric Functions, Differential Reduction.

INTRODUCTION

An essential part of theoretical physics has been the search for mathematical models that may explain complicated physical events in a simpler way. Feynman diagrams are one of the most effective frameworks for quantum field theory particle interactions, both visually and numerically. Applying generalised hypergeometric functions is one way to reduce the complexity of these computations in mathematics. These complex functions may be reduced to more comprehensible forms using differential reduction methods, which allows them to be applied to Feynman diagrams more easily. Applying differential reductions to simplify calculations and improve our knowledge of particle interactions is the main goal of this work, which mostly addresses the one-variable scenario. Our goal in taking this approach is to help scientists better understand and analyse the most basic processes in the universe by connecting theoretical concepts with real-world applications in physics.

BACKGROUND OF THE STUDY

Both mathematics and physics have benefited much from the dynamic relationship between the two disciplines. Because of their widespread use in solving complicated integrals and differential equations, hypergeometric functions have distinguished themselves among the many mathematical tools used in theoretical physics. The study of Feynman diagrams, statistical mechanics, quantum mechanics, and other fields rely heavily on these functions, which generalise the classical hypergeometric function. Midway through the twentieth century, Richard Feynman introduced the world to Feynman diagrams, which have since transformed how physicists think about and calculate interactions in quantum field theory. By simplifying mathematical formulas into pictorial representations, these diagrams illustrate the perturbative contributions to particle interactions. On the other hand, even the most advanced mathematical methods typically fail when faced with the complex integrals required to compute these diagrams.

An effective strategy for simplifying and solving the integrals linked to Feynman diagrams is the use of generalised hypergeometric functions in this setting. The flexibility and usefulness of generalised hypergeometric functions in mathematical physics are increased since they include a larger range of parameters and variables than typical hypergeometric functions. They are crucial in contemporary theoretical physics due to the fact that their reduction methods and differential characteristics may greatly simplify the assessment of Feynman integrals. This research investigates, for the one-variable situation in particular, how these sophisticated mathematical functions might be applied to the structure of Feynman diagrams. This study seeks to clarify and simplify the complicated calculations in Feynman diagram analysis by using the differential reduction methods of generalised hypergeometric functions. Physicists and mathematicians may benefit from a deeper knowledge of quantum interactions and from expanding their mathematical toolbox via the investigation of this technique.

THE PURPOSE OF THE RESEARCH

This research aims to investigate, for the one-variable situation in particular, the relevance and utility of differential reduction techniques applied to generalised hypergeometric functions in the framework of Feynman diagrams. This requires looking at how these mathematical tools may make the representation and computation of

Feynman diagrams easier, which are fundamental in particle physics and quantum field theory. The study's goal is to provide light on the mathematical principles that underlie the one-variable situation so that complicated physical calculations may be simplified.

LITERATURE REVIEW

Research into Feynman diagrams is essential to perturbative calculations in high-energy physics and quantum field theory (QFT). Presented by Richard Feynman in the 1940s, these diagrams provide a visual and calculative means of comprehending particle interactions, simplifying intricate integrals. The usage of generalised hypergeometric functions is one of the methods that have been devised throughout the years for evaluating these integrals.

A wide set of special functions that generalise the ordinary hypergeometric functions are generalised hypergeometric functions, abbreviated as kCc. They are able to characterise a vast array of mathematical physics phenomena and are characterised by series representations. One way to simplify the integrals by turning them into differential equations is to apply these functions to Feynman diagrams using differential reduction.

Gauss, Kummer, and Riemann were among the early mathematicians whose works established the basics of hypergeometric functions by analysing their characteristics and finding solutions to them. Later on, especially with the introduction of QFT, their physical significance became apparent. These functions were useful in theoretical physics because they could solve physical-related differential equations.

It wasn't until the middle of the twentieth century that hypergeometric functions were used to reduce Feynman integrals. Erdélyi and Bateman were among the researchers who contributed to the Bateman Manuscript Project, which included integrals and characteristics of hypergeometric functions, and who furthered the function's applications. The foundation for future applications in QFT was laid by their work. The differential reduction method, as applied to one-variable scenarios, is to write Feynman integrals as solutions to differential equations using generalised hypergeometric functions as the variables. Mathematicians and physicists who studied the relationships between QFT and special functions made substantial contributions to the method's methodical development in the 1970s and 1980s. Through the use of state-of-the-art computer resources and symbolic algebra systems, these methods have recently been improved upon. In order to evaluate Feynman diagrams more efficiently and accurately, researchers have created algorithms to automate the differential reduction process. Because of the exponential growth in complexity, these developments are of the utmost importance for computations requiring multi-loop diagrams. Additionally, beyond one-variable instances, the differential reduction technique has been expanded to more generic applications. An improved approach to solving Feynman integrals in higher dimensions may be found in the theory of multivariable hypergeometric functions and the differential equations that go along with them. Complex interactions in particle physics cannot be understood without this extension. The constant need for quick and accurate computing techniques in QFT keeps researchers busy studying the relationship between hypergeometric functions and Feynman diagrams. Insights and methods from the one-variable scenario guide the development of increasingly advanced applications.

Overall, a major step forward in the assessment of particle interaction integrals has been made by differentially reducing generalised hypergeometric functions to Feynman diagrams. This method is still an important tool for theoretical physics, as it is based on the long history of hypergeometric functions and is driven by current computing tools. Our knowledge of quantum field theory and its uses in high-energy physics stands to benefit from these approaches' ongoing growth and improvement.

RESEARCH QUESTIONS

1. What is the best way to reduce Feynman diagrams in the one-variable situation using differential reduction of generalised hypergeometric functions?

METHODOLOGY

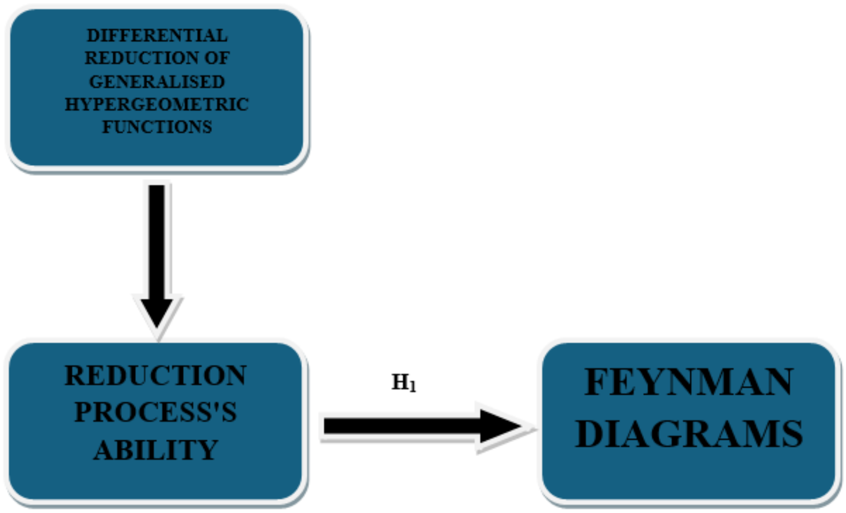
Research Design: By gathering numerical data on variables and feeding them into statistical models, quantitative research aims to discover statistically significant correlations between variables. Quantitative study ultimately aims to get a greater knowledge of society. Concerning human-related subjects, quantitative methods are often used by researchers. Visual representations of data, such tables and graphs, are common outcomes of quantitative research. Collecting and interpreting numerical information requires a systematic approach when dealing with quantitative data. Data averaging and forecasting are only a few of its many possible applications; others include studying relationships and expanding findings to larger populations. Qualitative studies, on the other hand, depend on in-depth interviews and observations (via text, video, or audio) and are therefore diametrically opposed to quantitative research. Quantitative research techniques are used in many academic disciplines. Economics, sociology, chemistry, biology, and marketing are all part of this category.

Sampling: After a successful pilot test with 20 Chinese consumers, the study was conducted with a final sample of 849 customers. Clients were selected at random and received a total of 900 questionnaires. None of the incomplete surveys were taken into account by the researcher.

Statistical Software: The statistical analysis was conducted using SPSS 25 and MS-Excel.

Statistical tools: The descriptive analysis helped the researchers to identify the core characteristics of the data. Factor analysis was used to assess validity.

CONCEPTUAL FRAMEWORK



RESULTS

There were 1000 questionnaires sent to the participants in all. Out of 975 surveys that were returned, 849 were evaluated using the Statistical Package for the Social Sciences (SPSS) version 25.0software.

- **Factor Analysis**

Verifying the latent component structure of a collection of measurement items is a common use of Factor Analysis (FA). It is often thought that the observable or measured variable scores are the result of latent (or unknown) causes. Accuracy analysis (FA) describes this method that relies on models. The main objective is to depict the interrelationships of the variables, taking into account the impact of measurement error and other factors that cannot be seen.

To determine whether data is appropriate for factor analysis, researchers might use the Kaiser-Meyer-Olkin (KMO) Method. Researchers looked at each model variable separately and the model as a whole to see whether sample size was enough. With the use of statistical measurements, we may determine the likelihood of a shared variance across several variables. It is common for data to become more suitable for factor analysis as the percentage is decreased.

The output from KMO is a number between 0 and 1. If the KMO number is between 0.8 and 1, it means that the sampling was sufficient.

If the KMO is less than 0.6, it means that the sample was insufficient and corrective action is needed. You may use your best judgement here; 0.5 has been used as an example by various writers, thus the range is 0.5–0.6.

The KMO indicates that the total correlations are small in comparison to the partial correlations if it's around zero. To reiterate, significant correlations significantly impede component analysis.

The standards that Kaiser has established for admission are as follows:

The standards that Kaiser has established for admission are as follows:

Extremely low, ranging from 0.050 to 0.059.

0.60-0.69 is not up to standard

Grades in the middle often range from 0.70 to 0.79.

Having a quality point score between 0.80 and 0.89.

The range from 0.90 to 1.00 is quite wide.

Table 1: KMO and Bartlett's Test[a]

KMO and Bartlett's Test[a]		
Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.868
Bartlett's Test of Sphericity	Approx. Chi-Square	4950.175
	df	190
	Sig.	.000
a. Based on correlations		

Claims made only for sampling are therefore legitimate. To make sure the correlation matrices were relevant, we ran them via Bartlett's Test of Sphericity. According to Kaiser-Meyer-Olkin, an adequate sample size is 0.868. The researchers used Bartlett's sphericity test and got a p-value of 0.00. An interesting discovery was made when Bartlett's sphericity test revealed that the correlation matrix is not an identity matrix.

Test for Hypothesis

It is common practice for scientific groups to "propose a hypothesis," or educated guess or assumption, before discussing the idea with peers and doing research to confirm or refute it. In order to formulate a testable hypothesis, the first stage in doing scientific research is to review the relevant literature. It turned out that the investigation's main premise was right. Providing a potential explanation for the observed phenomenon is as simple as making a "hypothesis" statement. It was necessary to formulate and test several hypotheses for the inquiry to be comprehensive.

DEPENDENT VARIABLE:

- **Feyman Diagrams**

A Feynman diagram is a useful tool for particle physicists to depict and calculate particle interactions. These schematics, drawn by physicist Richard Feynman to simplify complex computations, depict particles as lines with their intersections represented as vertices. The intersection of two lines represents an interaction, such as an electron absorbing or creating a photon. The lines themselves represent particles, with solid lines representing fermions (such as electrons) and wavy or dashed lines representing bosons (such as photons). In quantum field theories like QED, Feynman diagrams are essential for determining the likelihood of particle interactions and decay processes.

INDEPENDENT VARIABLE:

- **Differential Reduction of Generalised Hypergeometric Functions**

"Differential reduction of generated hypergeometric functions" is the way hypergeometric functions are simplified or changed by applying differential operators to them. As solutions to certain differential equations specified by ratios of polynomials, hypergeometric functions arise in several areas of mathematics and science. By applying operators to these functions, which are represented in terms of simpler or more fundamental hypergeometric functions, differential reduction often enables quicker evaluation, analysis, or computation. The simplification of solutions to complex integrals, series, or equations involving hypergeometric functions may depend on this step.

FACTOR:

- **Reduction Process's Ability**

The "reduction process's ability" refers to how well and efficiently a given process can simplify or reduce a system to its target state. Reduced numbers, simplified complexity, or reduced use of certain components, elements, or resources are all aspects of this capability. The degree to which a process can achieve reduction objectives is reflected by this term, which has applications in manufacturing, data processing, environmental management, and efficiency optimisation. The capacity to achieve reductions that are in line with the desired goals in a sustainable, accurate, and consistent manner is essential for enhancing performance, saving resources, decreasing costs, and minimising waste.

- **Relationship between Reduction Process's Ability and Feyman Diagrams**

How Feynman diagrams may reduce and visualise the complicated interactions and transformations in particle physics is what the "relationship between reduction process's ability and Feynman diagrams" is alluding to. Quantum field theory makes use of Feynman diagrams to illustrate decay, scattering, and particle interactions. Understanding and interpreting interaction probabilities, particle behaviour, and energy transfers within a process becomes much simpler with the help of these diagrams, which effectively provide a visual simplification of mathematical computations. To make the complicated algebraic equations needed to depict interactions understandable, the "reduction process's ability" in this setting emphasises the function of the diagrams.

On the basis of the above discussion, the researcher formulated the following hypothesis, which analysed the relationship between Creative Industries and Classroom Instruction.

H01: "There is no significant relationship between Reduction Process's Ability and Feyman Diagrams."

H1: "There is a significant relationship between Reduction Process's Ability and Feyman Diagrams."

Table.2: ANOVA test (H_1)

ANOVA					
Sum					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	42588.620	615	5575.517	1233.763	.000
Within Groups	377.770	233	5.356		
Total	42966.390	848			

"In this study, the result is significant. The value of F is 1233.763, which reaches significance with a p-value of .000 (which is less than the .05 alpha level). This means the H1: "There is a significant relationship between Reduction Process's Ability and Feyman Diagrams" is accepted and the null hypothesis is rejected."

DISCUSSION

This fascinating subject combines cutting-edge mathematical techniques with real-world applications in theoretical physics: the application of differential reduction to one-variable cases of generalised hypergeometric functions and Feynman diagrams. The complex structural features and variety of issues that generalised hypergeometric functions (kCc) may address make them significant in numerous domains. These functions originate from the calculation of loop integrals within the framework of Feynman diagrams, which are visual depictions of the perturbative contributions to the probability amplitude of quantum mechanical systems. A thorough understanding of the nature of generalised hypergeometric functions is necessary for comprehending their differential reduction. They have a series representation that converges under specified circumstances and generalise the classical hypergeometric function by introducing extra parameters. Quantum field theory (QFT) relies heavily on these functions because their parameters commonly match physical values in Feynman integrals. Fundamental to quantum field theory (QFT) are Feynman diagrams, which depict particle interactions as a network of vertices and edges. Evaluating integrals over loop momenta, a famously complicated operation, is often required for the calculation of amplitudes linked to these diagrams. Simplifying the computation procedure, these integrals may sometimes be written in terms of hypergeometric functions. The term "differential reduction" describes the process of simplifying a generalised hypergeometric function by applying differential operators to it. In order to simplify integrals seen in Feynman diagrams in a systematic way, this transformation takes use of the fact that hypergeometric functions satisfy differential equations. When dealing with a single complex variable, the emphasis is on hypergeometric functions in the one-variable situation. Even while this streamlines the analysis, it captures all the important aspects of the more general multi-variable case. These functions may be made more amenable to analytical or numerical treatment by using differential operators, which simplify the corresponding integrals. Not only does this method simplify the calculation of Feynman integrals for practical applications, but it also sheds light on the underlying structural features of the functions. In sum, a strong mathematical tool that improves our capacity to resolve complicated integrals in theoretical physics is the differential reduction of generalised hypergeometric functions to Feynman diagrams in the one-variable case. More efficient calculations are made possible by this link between sophisticated mathematics and physics, which also improves our knowledge of the basic mechanisms controlling particle interactions.

CONCLUSION

To sum up, Feynman diagrams in the one-variable situation may be effectively simplified and evaluated using the differential reduction of generalised hypergeometric functions. Complex integrals arising in Feynman diagrams may be more easily computed using this method, which combines cutting-edge mathematical methods with real-world applications in quantum field theory. In this case, generalised hypergeometric functions are useful because they can simplify complex diagrams by encapsulating all of the interdependencies and interactions. In addition to improving our computing skills, this technique helps us better grasp the mathematical structures at the heart of theoretical physics. Utilising the differential characteristics of these functions provides us a flexible instrument for dealing with various issues in quantum field theory, so adding to the continuous advancement of this fundamental branch of physics.

REFERENCES

1. Allen, L. (2008, April). The technology implications of „A Nation at Risk". Phi Delta Kappan, 608-610.
2. Brush, T., Ottenbreit-Leftwich, A., Kwon, K., & Karlin, M. (2020). Implementing socially relevant problem-based computer science curriculum at the elementary level: students' computer science knowledge and teachers' implementation needs. Journal of Computers in Mathematics and Science Teaching, 39(2), 109–123
3. Fields, D. A., Kafai, Y., Nakajima, T., Goode, J., & Margolis, J. (2018). Putting making into high school computer science classrooms: Promoting equity in teaching and learning with electronic textiles in exploring computer science. Equity & Excellence in Education, 51, 21–35. https://doi.org/10.1080/10665684.2018.1436998
4. García-Peñalvo, F. J., & Mendes, A. J. (2018). Exploring the computational thinking effects in pre-university education. Computers in Human Behavior, 80, 407–411. https://doi.org/10.1016/j.chb.2017.12.005
5. Hsu, T.-C., Chang, S.-C., & Hung, Iskrenovic-Momcilovic, O. (2019). Pair programming with scratch. Education and Information Technologies. https://doi.org/10.1007/s10639-019-09905-3
6. Hsu, T.-C., Chang, S.-C., & Hung, Y.-T. (2018). How to learn and how to teach computational thinking: Suggestions based on a review of the literature. Computers & Education, 126, 296–310. https://doi.org/10.1016/j.compedu.2018.07.004
7. Kwon, K., Lee, S. J., & Chung, J. (2018). Computational concepts reflected on Scratch programs. International Journal of Computer Science Education in Schools, 2(3), 1–15. https://doi.org/10.21585/ijcses.v2i3.33
8. Kwon, K., Leftwich, A., Brush, T., & Jeon, M. (2020). Effects of problem-based learning curriculum for computer science education in an elementary school. Paper presented at the AERA Annual Meeting, San Francisco, CA. http://tinyurl.com/wl5lak9.
9. Selwyn, N. (2008). Realising the potential of new technology? Assessing the legacy of new labour"s ICT agenda 1997-2007. Oxford Review of Education, 34, 701-712.